Exercises

1. The graph of f is given.

(a) Find each limit, or explain why it does not exist.

(i) $\lim_{x \to 2^+} f(x)$ (ii) $\lim_{x \to -3^+} f(x)$

(iii) $\lim_{x \to -3} f(x)$ (iv) $\lim_{x \to 4} f(x)$

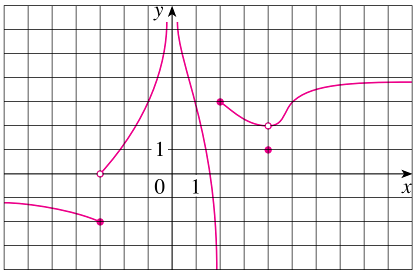
(v) $\lim_{x \to 0} f(x)$ (vi) $\lim_{x \to 2^-} f(x)$

(vii) $\lim_{x \to \infty} f(x)$ (viii) $\lim_{x \to -\infty} f(x)$

(b) State the equations of the horizontal asymptotes.

(c) State the equations of the vertical asymptotes.

(d) At what numbers is f discontinuous? Explain.



2. Sketch the graph of an example of a function f that satisfies all of the following conditions:

$\lim_{x \to -\infty} f(x) = -2$, $\lim_{x \to \infty} f(x) = 0$, $\lim_{x \to -3} f(x) = \infty$,

$\lim_{x \to 3^-} f(x) = -\infty$, $\lim_{x \to 3^+} f(x) = 2$,

f is continuous from the right at 3

3–20 Find the limit.

3. $\lim_{x \to 1} e^{x^3 - x}$

4. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

5. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

6. $\lim_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

7. $\lim_{h \to 0} \dfrac{(h - 1)^3 + 1}{h}$

8. $\lim_{t \to 2} \dfrac{t^2 - 4}{t^3 - 8}$

9. $\lim_{r \to 9} \dfrac{\sqrt{r}}{(r - 9)^4}$

10. $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|}$

11. $\lim_{u \to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u}$

12. $\lim_{x \to 3} \dfrac{\sqrt{x + 6} - x}{x^3 - 3x^2}$

13. $\lim_{x \to \infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$

14. $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$

15. $\lim_{x \to \pi^-} \ln(\sin x)$

16. $\lim_{x \to -\infty} \dfrac{1 - 2x^2 - x^4}{5 + x - 3x^4}$

17. $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x + 1} - x\right)$

18. $\lim_{x \to \infty} e^{x - x^2}$

19. $\lim_{x \to 0^+} \tan^{-1}(1/x)$

20. $\lim_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{1}{x^2 - 3x + 2}\right)$

21–22 Use graphs to discover the asymptotes of the curve. Then prove what you have discovered.

21. $y = \dfrac{\cos^2 x}{x^2}$

22. $y = \sqrt{x^2 + x + 1} - \sqrt{x^2 - x}$

23. If $2x - 1 \leqslant f(x) \leqslant x^2$ for $0 < x < 3$, find $\lim_{x \to 1} f(x)$.

24. Prove that $\lim_{x \to 0} x^2 \cos(1/x^2) = 0$.

25–28 Prove the statement using the precise definition of a limit.

25. $\lim_{x \to 2} (14 - 5x) = 4$

26. $\lim_{x \to 0} \sqrt[3]{x} = 0$

27. $\lim_{x \to 2} (x^2 - 3x) = -2$

28. $\lim_{x \to 4^+} \dfrac{2}{\sqrt{x - 4}} = \infty$

29. Let

$$f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leqslant x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases}$$

(a) Evaluate each limit, if it exists.

(i) $\lim_{x \to 0^+} f(x)$ (ii) $\lim_{x \to 0^-} f(x)$ (iii) $\lim_{x \to 0} f(x)$

(iv) $\lim_{x \to 3^-} f(x)$ (v) $\lim_{x \to 3^+} f(x)$ (vi) $\lim_{x \to 3} f(x)$

(b) Where is f discontinuous?

(c) Sketch the graph of f.

30. Let

$$g(x) = \begin{cases} 2x - x^2 & \text{if } 0 \leqslant x \leqslant 2 \\ 2 - x & \text{if } 2 < x \leqslant 3 \\ x - 4 & \text{if } 3 < x < 4 \\ \pi & \text{if } x \geqslant 4 \end{cases}$$

(a) For each of the numbers 2, 3, and 4, discover whether g is continuous from the left, continuous from the right, or continuous at the number.

(b) Sketch the graph of g.

Graphing calculator or computer required

31–32 Show that the function is continuous on its domain. State the domain.

31. $h(x) = xe^{\sin x}$

32. $g(x) = \dfrac{\sqrt{x^2 - 9}}{x^2 - 2}$

33–34 Use the Intermediate Value Theorem to show that there is a root of the equation in the given interval.

33. $x^5 - x^3 + 3x - 5 = 0, \quad (1, 2)$

34. $\cos\sqrt{x} = e^x - 2, \quad (0, 1)$

35. (a) Find the slope of the tangent line to the curve $y = 9 - 2x^2$ at the point $(2, 1)$.
(b) Find an equation of this tangent line.

36. Find equations of the tangent lines to the curve

$$y = \frac{2}{1 - 3x}$$

at the points with x-coordinates 0 and -1.

37. The displacement (in meters) of an object moving in a straight line is given by $s = 1 + 2t + \frac{1}{4}t^2$, where t is measured in seconds.
(a) Find the average velocity over each time period.
(i) $[1, 3]$ (ii) $[1, 2]$
(iii) $[1, 1.5]$ (iv) $[1, 1.1]$
(b) Find the instantaneous velocity when $t = 1$.

38. According to Boyle's Law, if the temperature of a confined gas is held fixed, then the product of the pressure P and the volume V is a constant. Suppose that, for a certain gas, $PV = 800$, where P is measured in pounds per square inch and V is measured in cubic inches.
(a) Find the average rate of change of P as V increases from 200 in^3 to 250 in^3.
(b) Express V as a function of P and show that the instantaneous rate of change of V with respect to P is inversely proportional to the square of P.

39. (a) Use the definition of a derivative to find $f'(2)$, where $f(x) = x^3 - 2x$.
(b) Find an equation of the tangent line to the curve $y = x^3 - 2x$ at the point $(2, 4)$.
(c) Illustrate part (b) by graphing the curve and the tangent line on the same screen.

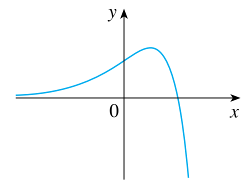
40. Find a function f and a number a such that

$$\lim_{h \to 0} \frac{(2 + h)^6 - 64}{h} = f'(a)$$

41. The total cost of repaying a student loan at an interest rate of r% per year is $C = f(r)$.
(a) What is the meaning of the derivative $f'(r)$? What are its units?
(b) What does the statement $f'(10) = 1200$ mean?
(c) Is $f'(r)$ always positive or does it change sign?

42–44 Trace or copy the graph of the function. Then sketch a graph of its derivative directly beneath.

42.



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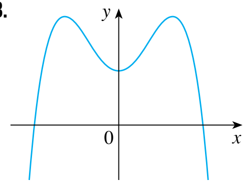

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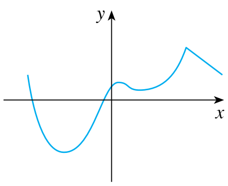
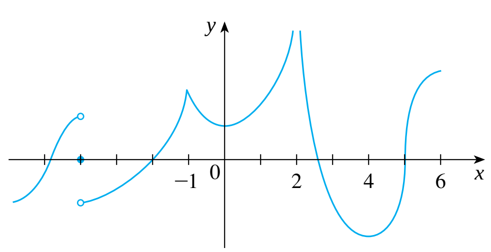


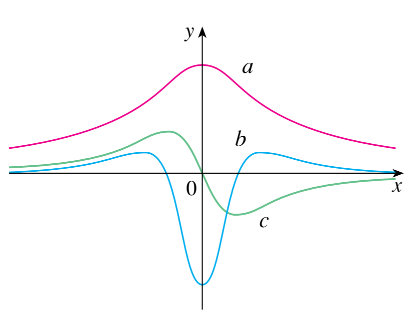
45. (a) If $f(x) = \sqrt{3 - 5x}$, use the definition of a derivative to find $f'(x)$.
(b) Find the domains of f and f'.
(c) Graph f and f' on a common screen. Compare the graphs to see whether your answer to part (a) is reasonable.

46. (a) Find the asymptotes of the graph of $f(x) = \dfrac{4 - x}{3 + x}$ and use them to sketch the graph.
(b) Use your graph from part (a) to sketch the graph of f'.
(c) Use the definition of a derivative to find $f'(x)$.
(d) Use a graphing device to graph f' and compare with your sketch in part (b).

47. The graph of f is shown. State, with reasons, the numbers at which f is not differentiable.



48. The figure shows the graphs of f, f', and f''. Identify each curve, and explain your choices.



49. Let $C(t)$ be the total value of US currency (coins and banknotes) in circulation at time t. The table gives values of this function from 1980 to 2000, as of September 30, in billions of dollars. Interpret and estimate the value of $C'(1990)$.

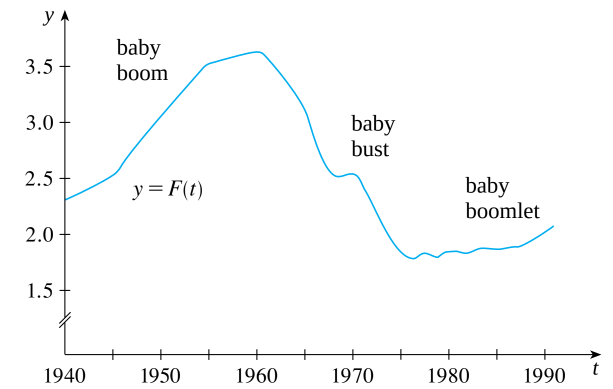
t	1980	1985	1990	1995	2000
$C(t)$	129.9	187.3	271.9	409.3	568.6

50. The *total fertility rate* at time t, denoted by $F(t)$, is an estimate of the average number of children born to each woman (assuming that current birth rates remain constant). The graph of the total fertility rate in the United States shows the fluctuations from 1940 to 1990.

(a) Estimate the values of $F'(1950)$, $F'(1965)$, and $F'(1987)$.

(b) What are the meanings of these derivatives?

(c) Can you suggest reasons for the values of these derivatives?



51. Suppose that $|f(x)| \leqslant g(x)$ for all x, where $\lim_{x\to a} g(x) = 0$. Find $\lim_{x\to a} f(x)$.

52. Let $f(x) = [\![x]\!] + [\![-x]\!]$.

(a) For what values of a does $\lim_{x\to a} f(x)$ exist?

(b) At what numbers is f discontinuous?