

2.1 Exercises

1. A tank holds 1000 gallons of water, which drains from the bottom of the tank in half an hour. The values in the table show the volume V of water remaining in the tank (in gallons) after t minutes.

t (min)	5	10	15	20	25	30
V (gal)	694	444	250	111	28	0

- (a) If P is the point $(15, 250)$ on the graph of V , find the slopes of the secant lines PQ when Q is the point on the graph with $t = 5, 10, 20, 25,$ and 30 .
- (b) Estimate the slope of the tangent line at P by averaging the slopes of two secant lines.
- (c) Use a graph of the function to estimate the slope of the tangent line at P . (This slope represents the rate at which the water is flowing from the tank after 15 minutes.)
2. A cardiac monitor is used to measure the heart rate of a patient after surgery. It compiles the number of heartbeats after t minutes. When the data in the table are graphed, the slope of the tangent line represents the heart rate in beats per minute.

t (min)	36	38	40	42	44
Heartbeats	2530	2661	2806	2948	3080

The monitor estimates this value by calculating the slope of a secant line. Use the data to estimate the patient's heart rate after 42 minutes using the secant line between the points with the given values of t .

- (a) $t = 36$ and $t = 42$ (b) $t = 38$ and $t = 42$
 (c) $t = 40$ and $t = 42$ (d) $t = 42$ and $t = 44$

What are your conclusions?

3. The point $P(2, -1)$ lies on the curve $y = 1/(1 - x)$.
- (a) If Q is the point $(x, 1/(1 - x))$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x :
- (i) 1.5 (ii) 1.9 (iii) 1.99 (iv) 1.999
 (v) 2.5 (vi) 2.1 (vii) 2.01 (viii) 2.001
- (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(2, -1)$.
- (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(2, -1)$.
4. The point $P(0.5, 0)$ lies on the curve $y = \cos \pi x$.
- (a) If Q is the point $(x, \cos \pi x)$, use your calculator to find the slope of the secant line PQ (correct to six decimal places) for the following values of x :
- (i) 0 (ii) 0.4 (iii) 0.49 (iv) 0.499
 (v) 1 (vi) 0.6 (vii) 0.51 (viii) 0.501
- (b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(0.5, 0)$.

- (c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(0.5, 0)$.
- (d) Sketch the curve, two of the secant lines, and the tangent line.

5. If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $y = 40t - 16t^2$.
- (a) Find the average velocity for the time period beginning when $t = 2$ and lasting
- (i) 0.5 second (ii) 0.1 second
 (iii) 0.05 second (iv) 0.01 second
- (b) Estimate the instantaneous velocity when $t = 2$.
6. If a rock is thrown upward on the planet Mars with a velocity of 10 m/s, its height in meters t seconds later is given by $y = 10t - 1.86t^2$.
- (a) Find the average velocity over the given time intervals:
- (i) $[1, 2]$ (ii) $[1, 1.5]$ (iii) $[1, 1.1]$
 (iv) $[1, 1.01]$ (v) $[1, 1.001]$
- (b) Estimate the instantaneous velocity when $t = 1$.

7. The table shows the position of a cyclist.

t (seconds)	0	1	2	3	4	5
s (meters)	0	1.4	5.1	10.7	17.7	25.8

- (a) Find the average velocity for each time period:
- (i) $[1, 3]$ (ii) $[2, 3]$ (iii) $[3, 5]$ (iv) $[3, 4]$
- (b) Use the graph of s as a function of t to estimate the instantaneous velocity when $t = 3$.
8. The displacement (in centimeters) of a particle moving back and forth along a straight line is given by the equation of motion $s = 2 \sin \pi t + 3 \cos \pi t$, where t is measured in seconds.
- (a) Find the average velocity during each time period:
- (i) $[1, 2]$ (ii) $[1, 1.1]$
 (iii) $[1, 1.01]$ (iv) $[1, 1.001]$
- (b) Estimate the instantaneous velocity of the particle when $t = 1$.
9. The point $P(1, 0)$ lies on the curve $y = \sin(10\pi/x)$.
- (a) If Q is the point $(x, \sin(10\pi/x))$, find the slope of the secant line PQ (correct to four decimal places) for $x = 2, 1.5, 1.4, 1.3, 1.2, 1.1, 0.5, 0.6, 0.7, 0.8,$ and 0.9 . Do the slopes appear to be approaching a limit?
- (b) Use a graph of the curve to explain why the slopes of the secant lines in part (a) are not close to the slope of the tangent line at P .
- (c) By choosing appropriate secant lines, estimate the slope of the tangent line at P .