

## 3.6 Exercises

1. Explain why the natural logarithmic function  $y = \ln x$  is used much more frequently in calculus than the other logarithmic functions  $y = \log_a x$ .

2–22 Differentiate the function.

2.  $f(x) = x \ln x - x$

3.  $f(x) = \sin(\ln x)$

5.  $f(x) = \ln \frac{1}{x}$

7.  $f(x) = \log_{10}(x^3 + 1)$

9.  $f(x) = \sin x \ln(5x)$

11.  $g(x) = \ln(x\sqrt{x^2 - 1})$

13.  $G(y) = \ln \frac{(2y + 1)^5}{\sqrt{y^2 + 1}}$

15.  $F(s) = \ln \ln s$

17.  $y = \tan[\ln(ax + b)]$

19.  $y = \ln(e^{-x} + xe^{-x})$

21.  $y = 2x \log_{10} \sqrt{x}$

4.  $f(x) = \ln(\sin^2 x)$

6.  $y = \frac{1}{\ln x}$

8.  $f(x) = \log_5(xe^x)$

10.  $f(u) = \frac{u}{1 + \ln u}$

12.  $h(x) = \ln(x + \sqrt{x^2 - 1})$

14.  $g(r) = r^2 \ln(2r + 1)$

16.  $y = \ln |1 + t - t^3|$

18.  $y = \ln |\cos(\ln x)|$

20.  $H(z) = \ln \sqrt{\frac{a^2 - z^2}{a^2 + z^2}}$

22.  $y = \log_2(e^{-x} \cos \pi x)$

23–26 Find  $y'$  and  $y''$ .

23.  $y = x^2 \ln(2x)$

24.  $y = \frac{\ln x}{x^2}$

25.  $y = \ln(x + \sqrt{1 + x^2})$

26.  $y = \ln(\sec x + \tan x)$

27–30 Differentiate  $f$  and find the domain of  $f$ .

27.  $f(x) = \frac{x}{1 - \ln(x - 1)}$

28.  $f(x) = \sqrt{2 + \ln x}$

29.  $f(x) = \ln(x^2 - 2x)$

30.  $f(x) = \ln \ln \ln x$


31. If  $f(x) = \frac{\ln x}{x^2}$ , find  $f'(1)$ .


32. If  $f(x) = \ln(1 + e^{2x})$ , find  $f'(0)$ .

33–34 Find an equation of the tangent line to the curve at the given point.

33.  $y = \ln(x^2 - 3x + 1)$ ,  $(3, 0)$

34.  $y = x^2 \ln x$ ,  $(1, 0)$

 35. If  $f(x) = \sin x + \ln x$ , find  $f'(x)$ . Check that your answer is reasonable by comparing the graphs of  $f$  and  $f'$ .

 36. Find equations of the tangent lines to the curve  $y = (\ln x)/x$  at the points  $(1, 0)$  and  $(e, 1/e)$ . Illustrate by graphing the curve and its tangent lines.

37. Let  $f(x) = cx + \ln(\cos x)$ . For what value of  $c$  is  $f'(\pi/4) = 6$ ?

38. Let  $f(x) = \log_a(3x^2 - 2)$ . For what value of  $a$  is  $f'(1) = 3$ ?

39–50 Use logarithmic differentiation to find the derivative of the function.

39.  $y = (x^2 + 2)^2(x^4 + 4)^4$

40.  $y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1}$

41.  $y = \sqrt{\frac{x - 1}{x^4 + 1}}$

42.  $y = \sqrt{x} e^{x^2 - x} (x + 1)^{2/3}$

43.  $y = x^x$

44.  $y = x^{\cos x}$

45.  $y = x^{\sin x}$

46.  $y = \sqrt{x}^x$

47.  $y = (\cos x)^x$

48.  $y = (\sin x)^{\ln x}$

49.  $y = (\tan x)^{1/x}$

50.  $y = (\ln x)^{\cos x}$

51. Find  $y'$  if  $y = \ln(x^2 + y^2)$ .

52. Find  $y'$  if  $x^y = y^x$ .

53. Find a formula for  $f^{(n)}(x)$  if  $f(x) = \ln(x - 1)$ .

54. Find  $\frac{d^9}{dx^9}(x^8 \ln x)$ .

55. Use the definition of derivative to prove that

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x)}{x} = 1$$

56. Show that  $\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x$  for any  $x > 0$ .