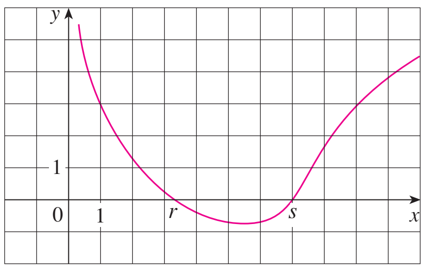
4.8 Exercises

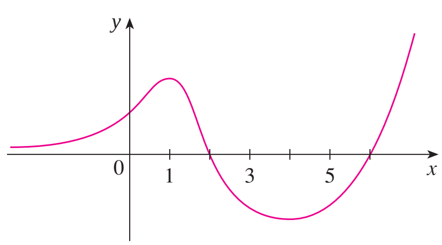
1. The figure shows the graph of a function f. Suppose that Newton's method is used to approximate the root r of the equation $f(x) = 0$ with initial approximation $x_1 = 1$.
(a) Draw the tangent lines that are used to find x_2 and x_3, and estimate the numerical values of x_2 and x_3.
(b) Would $x_1 = 5$ be a better first approximation? Explain.



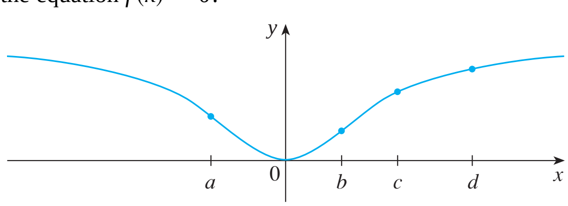
2. Follow the instructions for Exercise 1(a) but use $x_1 = 9$ as the starting approximation for finding the root s.

3. Suppose the tangent line to the curve $y = f(x)$ at the point $(2, 5)$ has the equation $y = 9 - 2x$. If Newton's method is used to locate a root of the equation $f(x) = 0$ and the initial approximation is $x_1 = 2$, find the second approximation x_2.

4. For each initial approximation, determine graphically what happens if Newton's method is used for the function whose graph is shown.
(a) $x_1 = 0$ (b) $x_1 = 1$ (c) $x_1 = 3$
(d) $x_1 = 4$ (e) $x_1 = 5$



5. For which of the initial approximations $x_1 = a$, b, c, and d do you think Newton's method will work and lead to the root of the equation $f(x) = 0$?



6–8 Use Newton's method with the specified initial approximation x_1 to find x_3, the third approximation to the root of the given equation. (Give your answer to four decimal places.)

6. $\frac{1}{3}x^3 + \frac{1}{2}x^2 + 3 = 0, \quad x_1 = -3$

7. $x^5 - x - 1 = 0, \quad x_1 = 1$

8. $x^7 + 4 = 0, \quad x_1 = -1$

9. Use Newton's method with initial approximation $x_1 = -1$ to find x_2, the second approximation to the root of the equation $x^3 + x + 3 = 0$. Explain how the method works by first graphing the function and its tangent line at $(-1, 1)$.

10. Use Newton's method with initial approximation $x_1 = 1$ to find x_2, the second approximation to the root of the equation $x^4 - x - 1 = 0$. Explain how the method works by first graphing the function and its tangent line at $(1, -1)$.

11–12 Use Newton's method to approximate the given number correct to eight decimal places.

11. $\sqrt[5]{20}$

12. $\sqrt[100]{100}$

13–16 Use Newton's method to approximate the indicated root of the equation correct to six decimal places.

13. The root of $x^4 - 2x^3 + 5x^2 - 6 = 0$ in the interval $[1, 2]$

14. The root of $2.2x^5 - 4.4x^3 + 1.3x^2 - 0.9x - 4.0 = 0$ in the interval $[-2, -1]$

15. The negative root of $e^x = 4 - x^2$

16. The positive root of $3 \sin x = x$

17–22 Use Newton's method to find all roots of the equation correct to six decimal places.

17. $3 \cos x = x + 1$

18. $\sqrt{x + 1} = x^2 - x$

19. $(x - 2)^2 = \ln x$

20. $\dfrac{1}{x} = 1 + x^3$

21. $x^3 = \tan^{-1} x$

22. $\sin x = x^2 - 2$

23–28 Use Newton's method to find all the roots of the equation correct to eight decimal places. Start by drawing a graph to find initial approximations.

23. $x^6 - x^5 - 6x^4 - x^2 + x + 10 = 0$

24. $x^5 - 3x^4 + x^3 - x^2 - x + 6 = 0$

25. $\dfrac{x}{x^2 + 1} = \sqrt{1 - x}$

26. $\cos(x^2 - x) = x^4$

27. $4e^{-x^2} \sin x = x^2 - x + 1$

28. $e^{\arctan x} = \sqrt{x^3 + 1}$

29. (a) Apply Newton's method to the equation $x^2 - a = 0$ to derive the following square-root algorithm (used by the ancient Babylonians to compute $\sqrt{a}$):

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$$

Graphing calculator or computer required   **1.** Homework Hints available at stewartcalculus.com

(b) Use part (a) to compute $\sqrt{1000}$ correct to six decimal places.

30. (a) Apply Newton's method to the equation $1/x - a = 0$ to derive the following reciprocal algorithm:

$$x_{n+1} = 2x_n - ax_n^2$$

(This algorithm enables a computer to find reciprocals without actually dividing.)

(b) Use part (a) to compute $1/1.6984$ correct to six decimal places.

31. Explain why Newton's method doesn't work for finding the root of the equation $x^3 - 3x + 6 = 0$ if the initial approximation is chosen to be $x_1 = 1$.

32. (a) Use Newton's method with $x_1 = 1$ to find the root of the equation $x^3 - x = 1$ correct to six decimal places.

(b) Solve the equation in part (a) using $x_1 = 0.6$ as the initial approximation.

(c) Solve the equation in part (a) using $x_1 = 0.57$. (You definitely need a programmable calculator for this part.)

(d) Graph $f(x) = x^3 - x - 1$ and its tangent lines at $x_1 = 1$, 0.6, and 0.57 to explain why Newton's method is so sensitive to the value of the initial approximation.

33. Explain why Newton's method fails when applied to the equation $\sqrt[3]{x} = 0$ with any initial approximation $x_1 \neq 0$. Illustrate your explanation with a sketch.

34. If

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geqslant 0 \\ -\sqrt{-x} & \text{if } x < 0 \end{cases}$$

then the root of the equation $f(x) = 0$ is $x = 0$. Explain why Newton's method fails to find the root no matter which initial approximation $x_1 \neq 0$ is used. Illustrate your explanation with a sketch.

35. (a) Use Newton's method to find the critical numbers of the function $f(x) = x^6 - x^4 + 3x^3 - 2x$ correct to six decimal places.

(b) Find the absolute minimum value of f correct to four decimal places.

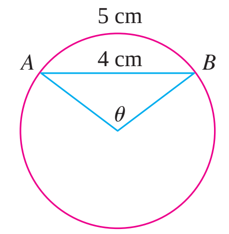
36. Use Newton's method to find the absolute maximum value of the function $f(x) = x\cos x$, $0 \leqslant x \leqslant \pi$, correct to six decimal places.

37. Use Newton's method to find the coordinates of the inflection point of the curve $y = x^2 \sin x$, $0 \leqslant x \leqslant \pi$, correct to six decimal places.

38. Of the infinitely many lines that are tangent to the curve $y = -\sin x$ and pass through the origin, there is one that has the largest slope. Use Newton's method to find the slope of that line correct to six decimal places.

39. Use Newton's method to find the coordinates, correct to six decimal places, of the point on the parabola $y = (x - 1)^2$ that is closest to the origin.

40. In the figure, the length of the chord AB is 4 cm and the length of the arc AB is 5 cm. Find the central angle θ, in radians, correct to four decimal places. Then give the answer to the nearest degree.



41. A car dealer sells a new car for \$18,000. He also offers to sell the same car for payments of \$375 per month for five years. What monthly interest rate is this dealer charging?

To solve this problem you will need to use the formula for the present value A of an annuity consisting of n equal payments of size R with interest rate i per time period:

$$A = \frac{R}{i}[1 - (1 + i)^{-n}]$$

Replacing i by x, show that

$$48x(1 + x)^{60} - (1 + x)^{60} + 1 = 0$$

Use Newton's method to solve this equation.

42. The figure shows the sun located at the origin and the earth at the point (1, 0). (The unit here is the distance between the centers of the earth and the sun, called an *astronomical unit:* 1 AU $\approx 1.496 \times 10^8$ km.) There are five locations L_1, L_2, L_3, L_4, and L_5 in this plane of rotation of the earth about the sun where a satellite remains motionless with respect to the earth because the forces acting on the satellite (including the gravitational attractions of the earth and the sun) balance each other. These locations are called *libration points.* (A solar research satellite has been placed at one of these libration points.) If m_1 is the mass of the sun, m_2 is the mass of the earth, and $r = m_2/(m_1 + m_2)$, it turns out that the x-coordinate of L_1 is the unique root of the fifth-degree equation

$$p(x) = x^5 - (2 + r)x^4 + (1 + 2r)x^3 - (1 - r)x^2 + 2(1 - r)x + r - 1 = 0$$

and the x-coordinate of L_2 is the root of the equation

$$p(x) - 2rx^2 = 0$$

Using the value $r \approx 3.04042 \times 10^{-6}$, find the locations of the libration points (a) L_1 and (b) L_2.