Figure 8 shows the interpretation of the arc length function in Example 4. Figure 9 shows the graph of this arc length function. Why is $s(x)$ negative when x is less than 1?

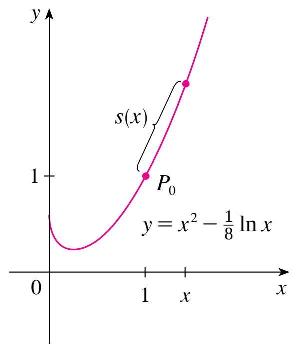


FIGURE 8

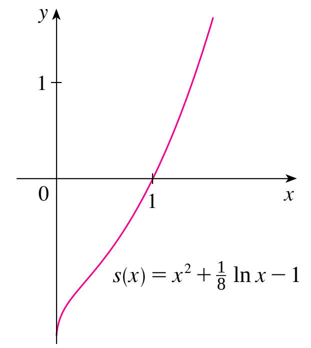


FIGURE 9

8.1 Exercises

1. Use the arc length formula [3] to find the length of the curve $y = 2x - 5$, $-1 \leqslant x \leqslant 3$. Check your answer by noting that the curve is a line segment and calculating its length by the distance formula.

2. Use the arc length formula to find the length of the curve $y = \sqrt{2 - x^2}$, $0 \leqslant x \leqslant 1$. Check your answer by noting that the curve is part of a circle.

3–6 Set up an integral that represents the length of the curve. Then use your calculator to find the length correct to four decimal places.

3. $y = \sin x, \quad 0 \leqslant x \leqslant \pi$

4. $y = xe^{-x}, \quad 0 \leqslant x \leqslant 2$

5. $x = \sqrt{y} - y, \quad 1 \leqslant y \leqslant 4$

6. $x = y^2 - 2y, \quad 0 \leqslant y \leqslant 2$

7–18 Find the exact length of the curve.

7. $y = 1 + 6x^{3/2}, \quad 0 \leqslant x \leqslant 1$

8. $y^2 = 4(x + 4)^3, \quad 0 \leqslant x \leqslant 2, \quad y > 0$

9. $y = \dfrac{x^3}{3} + \dfrac{1}{4x}, \quad 1 \leqslant x \leqslant 2$

10. $x = \dfrac{y^4}{8} + \dfrac{1}{4y^2}, \quad 1 \leqslant y \leqslant 2$

11. $x = \frac{1}{3}\sqrt{y}\,(y - 3), \quad 1 \leqslant y \leqslant 9$

12. $y = \ln(\cos x), \quad 0 \leqslant x \leqslant \pi/3$

13. $y = \ln(\sec x), \quad 0 \leqslant x \leqslant \pi/4$

14. $y = 3 + \frac{1}{2}\cosh 2x, \quad 0 \leqslant x \leqslant 1$

15. $y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \leqslant x \leqslant 2$

16. $y = \sqrt{x - x^2} + \sin^{-1}\left(\sqrt{x}\right)$

17. $y = \ln(1 - x^2), \quad 0 \leqslant x \leqslant \frac{1}{2}$

18. $y = 1 - e^{-x}, \quad 0 \leqslant x \leqslant 2$

19–20 Find the length of the arc of the curve from point P to point Q.

19. $y = \frac{1}{2}x^2, \quad P\left(-1, \frac{1}{2}\right), \quad Q\left(1, \frac{1}{2}\right)$

20. $x^2 = (y - 4)^3, \quad P(1, 5), \quad Q(8, 8)$

21–22 Graph the curve and visually estimate its length. Then use your calculator to find the length correct to four decimal places.

21. $y = x^2 + x^3, \quad 1 \leqslant x \leqslant 2$

22. $y = x + \cos x, \quad 0 \leqslant x \leqslant \pi/2$

23–26 Use Simpson's Rule with $n = 10$ to estimate the arc length of the curve. Compare your answer with the value of the integral produced by your calculator.

23. $y = x \sin x, \quad 0 \leqslant x \leqslant 2\pi$

24. $y = \sqrt[3]{x}, \quad 1 \leqslant x \leqslant 6$

25. $y = \ln(1 + x^3), \quad 0 \leqslant x \leqslant 5$

26. $y = e^{-x^2}, \quad 0 \leqslant x \leqslant 2$

27. (a) Graph the curve $y = x\sqrt[3]{4 - x}$, $0 \leqslant x \leqslant 4$.
(b) Compute the lengths of inscribed polygons with $n = 1, 2,$ and 4 sides. (Divide the interval into equal subintervals.) Illustrate by sketching these polygons (as in Figure 6).
(c) Set up an integral for the length of the curve.
(d) Use your calculator to find the length of the curve to four decimal places. Compare with the approximations in part (b).

Graphing calculator or computer required   CAS Computer algebra system required   **1.** Homework Hints available at stewartcalculus.com

28. Repeat Exercise 27 for the curve

$$y = x + \sin x \qquad 0 \leqslant x \leqslant 2\pi$$

CAS **29.** Use either a computer algebra system or a table of integrals to find the *exact* length of the arc of the curve $y = \ln x$ that lies between the points $(1, 0)$ and $(2, \ln 2)$.

CAS **30.** Use either a computer algebra system or a table of integrals to find the *exact* length of the arc of the curve $y = x^{4/3}$ that lies between the points $(0, 0)$ and $(1, 1)$. If your CAS has trouble evaluating the integral, make a substitution that changes the integral into one that the CAS can evaluate.

31. Sketch the curve with equation $x^{2/3} + y^{2/3} = 1$ and use symmetry to find its length.

32. (a) Sketch the curve $y^3 = x^2$.
(b) Use Formulas 3 and 4 to set up two integrals for the arc length from $(0, 0)$ to $(1, 1)$. Observe that one of these is an improper integral and evaluate both of them.
(c) Find the length of the arc of this curve from $(-1, 1)$ to $(8, 4)$.

33. Find the arc length function for the curve $y = 2x^{3/2}$ with starting point $P_0(1, 2)$.

34. (a) Find the arc length function for the curve $y = \ln(\sin x)$, $0 < x < \pi$, with starting point $(\pi/2, 0)$.
(b) Graph both the curve and its arc length function on the same screen.

35. Find the arc length function for the curve $y = \sin^{-1}x + \sqrt{1 - x^2}$ with starting point $(0, 1)$.

36. A steady wind blows a kite due west. The kite's height above ground from horizontal position $x = 0$ to $x = 80$ ft is given by $y = 150 - \frac{1}{40}(x - 50)^2$. Find the distance traveled by the kite.

37. A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

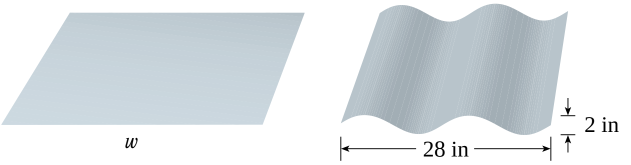
until it hits the ground, where y is its height above the ground and x is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter.

38. The Gateway Arch in St. Louis (see the photo on page 259) was constructed using the equation

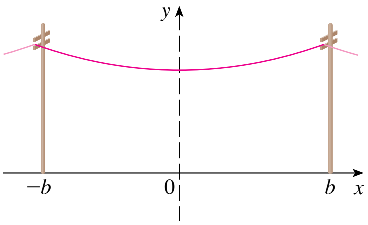
$$y = 211.49 - 20.96 \cosh 0.03291765x$$

for the central curve of the arch, where x and y are measured in meters and $|x| \leqslant 91.20$. Set up an integral for the length of the arch and use your calculator to estimate the length correct to the nearest meter.

39. A manufacturer of corrugated metal roofing wants to produce panels that are 28 in. wide and 2 in. thick by processing flat sheets of metal as shown in the figure. The profile of the roofing takes the shape of a sine wave. Verify that the sine curve has equation $y = \sin(\pi x/7)$ and find the width w of a flat metal sheet that is needed to make a 28-inch panel. (Use your calculator to evaluate the integral correct to four significant digits.)



40. (a) The figure shows a telephone wire hanging between two poles at $x = -b$ and $x = b$. It takes the shape of a catenary with equation $y = c + a \cosh(x/a)$. Find the length of the wire.
(b) Suppose two telephone poles are 50 ft apart and the length of the wire between the poles is 51 ft. If the lowest point of the wire must be 20 ft above the ground, how high up on each pole should the wire be attached?



41. Find the length of the curve

$$y = \int_1^x \sqrt{t^3 - 1}\, dt \qquad 1 \leqslant x \leqslant 4$$

42. The curves with equations $x^n + y^n = 1$, $n = 4, 6, 8, \ldots$, are called **fat circles**. Graph the curves with $n = 2, 4, 6, 8$, and 10 to see why. Set up an integral for the length L_{2k} of the fat circle with $n = 2k$. Without attempting to evaluate this integral, state the value of $\lim_{k \to \infty} L_{2k}$.