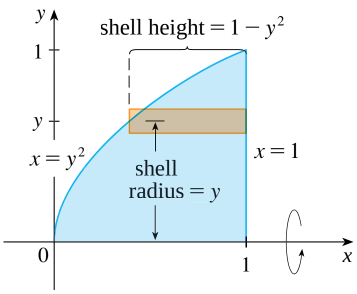


FIGURE 9

SOLUTION This problem was solved using disks in Example 2 in Section 6.2. To use shells we relabel the curve $y = \sqrt{x}$ (in the figure in that example) as $x = y^2$ in Figure 9. For rotation about the x-axis we see that a typical shell has radius y, circumference $2\pi y$, and height $1 - y^2$. So the volume is

$$V = \int_0^1 (2\pi y)(1 - y^2)\,dy = 2\pi \int_0^1 (y - y^3)\,dy$$

$$= 2\pi \left[\frac{y^2}{2} - \frac{y^4}{4} \right]_0^1 = \frac{\pi}{2}$$

In this problem the disk method was simpler.

V EXAMPLE 4 Find the volume of the solid obtained by rotating the region bounded by $y = x - x^2$ and $y = 0$ about the line $x = 2$.

SOLUTION Figure 10 shows the region and a cylindrical shell formed by rotation about the line $x = 2$. It has radius $2 - x$, circumference $2\pi(2 - x)$, and height $x - x^2$.

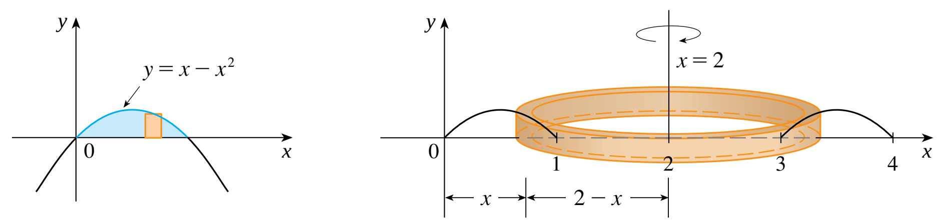


FIGURE 10

The volume of the given solid is

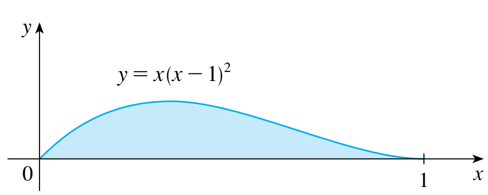
$$V = \int_0^1 2\pi(2 - x)(x - x^2)\,dx$$

$$= 2\pi \int_0^1 (x^3 - 3x^2 + 2x)\,dx$$

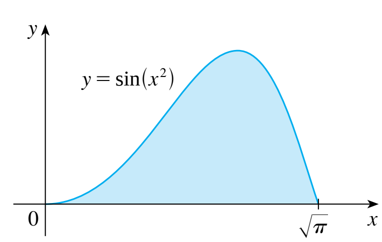
$$= 2\pi \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1 = \frac{\pi}{2}$$

6.3 Exercises

1. Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Explain why it is awkward to use slicing to find the volume V of S. Sketch a typical approximating shell. What are its circumference and height? Use shells to find V.



2. Let S be the solid obtained by rotating the region shown in the figure about the y-axis. Sketch a typical cylindrical shell and find its circumference and height. Use shells to find the volume of S. Do you think this method is preferable to slicing? Explain.



Graphing calculator or computer required   CAS Computer algebra system required   **1.** Homework Hints available at stewartcalculus.com

3–7 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the y-axis.

3. $y = \sqrt[3]{x}, \quad y = 0, \quad x = 1$

4. $y = x^3, \quad y = 0, \quad x = 1, \quad x = 2$

5. $y = e^{-x^2}, \quad y = 0, \quad x = 0, \quad x = 1$

6. $y = 4x - x^2, \quad y = x$

7. $y = x^2, \quad y = 6x - 2x^2$

---

8. Let V be the volume of the solid obtained by rotating about the y-axis the region bounded by $y = \sqrt{x}$ and $y = x^2$. Find V both by slicing and by cylindrical shells. In both cases draw a diagram to explain your method.

9–14 Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the given curves about the x-axis.

9. $xy = 1, \quad x = 0, \quad y = 1, \quad y = 3$

10. $y = \sqrt{x}, \quad x = 0, \quad y = 2$

11. $y = x^3, \quad y = 8, \quad x = 0$

12. $x = 4y^2 - y^3, \quad x = 0$

13. $x = 1 + (y - 2)^2, \quad x = 2$

14. $x + y = 3, \quad x = 4 - (y - 1)^2$

15–20 Use the method of cylindrical shells to find the volume generated by rotating the region bounded by the given curves about the specified axis.

15. $y = x^4,\ y = 0,\ x = 1; \quad$ about $x = 2$

16. $y = \sqrt{x},\ y = 0,\ x = 1; \quad$ about $x = -1$

17. $y = 4x - x^2,\ y = 3; \quad$ about $x = 1$

18. $y = x^2,\ y = 2 - x^2; \quad$ about $x = 1$

19. $y = x^3,\ y = 0,\ x = 1; \quad$ about $y = 1$

20. $x = y^2 + 1,\ x = 2; \quad$ about $y = -2$

21–26

(a) Set up an integral for the volume of the solid obtained by rotating the region bounded by the given curve about the specified axis.

(b) Use your calculator to evaluate the integral correct to five decimal places.

21. $y = xe^{-x},\ y = 0,\ x = 2; \quad$ about the y-axis

22. $y = \tan x,\ y = 0,\ x = \pi/4; \quad$ about $x = \pi/2$

23. $y = \cos^4 x,\ y = -\cos^4 x,\ -\pi/2 \leqslant x \leqslant \pi/2; \quad$ about $x = \pi$

24. $y = x,\ y = 2x/(1 + x^3); \quad$ about $x = -1$

25. $x = \sqrt{\sin y},\ 0 \leqslant y \leqslant \pi,\ x = 0; \quad$ about $y = 4$

26. $x^2 - y^2 = 7,\ x = 4; \quad$ about $y = 5$

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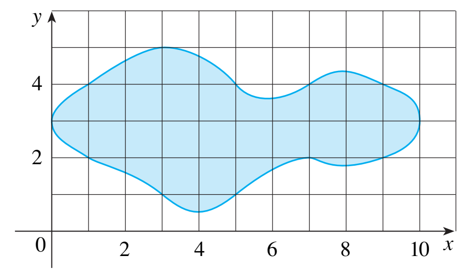
27. Use the Midpoint Rule with $n = 5$ to estimate the volume obtained by rotating about the y-axis the region under the curve $y = \sqrt{1 + x^3}$, $0 \leqslant x \leqslant 1$.

28. If the region shown in the figure is rotated about the y-axis to form a solid, use the Midpoint Rule with $n = 5$ to estimate the volume of the solid.



29–32 Each integral represents the volume of a solid. Describe the solid.

29. $\displaystyle\int_0^3 2\pi x^5\, dx$

30. $\displaystyle 2\pi \int_0^2 \frac{y}{1 + y^2}\, dy$

31. $\displaystyle\int_0^1 2\pi(3 - y)(1 - y^2)\, dy$

32. $\displaystyle\int_0^{\pi/4} 2\pi(\pi - x)(\cos x - \sin x)\, dx$

33–34 Use a graph to estimate the x-coordinates of the points of intersection of the given curves. Then use this information and your calculator to estimate the volume of the solid obtained by rotating about the y-axis the region enclosed by these curves.

33. $y = e^x, \quad y = \sqrt{x} + 1$

34. $y = x^3 - x + 1, \quad y = -x^4 + 4x - 1$

CAS **35–36** Use a computer algebra system to find the exact volume of the solid obtained by rotating the region bounded by the given curves about the specified line.

35. $y = \sin^2 x,\ y = \sin^4 x,\ 0 \leqslant x \leqslant \pi; \quad$ about $x = \pi/2$

36. $y = x^3 \sin x,\ y = 0,\ 0 \leqslant x \leqslant \pi; \quad$ about $x = -1$

37–43 The region bounded by the given curves is rotated about the specified axis. Find the volume of the resulting solid by any method.

37. $y = -x^2 + 6x - 8,\ y = 0; \quad$ about the y-axis

38. $y = -x^2 + 6x - 8,\ y = 0; \quad$ about the x-axis

39. $y^2 - x^2 = 1,\ y = 2; \quad$ about the x-axis

40. $y^2 - x^2 = 1,\ y = 2; \quad$ about the y-axis

41. $x^2 + (y - 1)^2 = 1$; about the y-axis

42. $x = (y - 3)^2$, $x = 4$; about $y = 1$

43. $x = (y - 1)^2$, $x - y = 1$; about $x = -1$

44. Let T be the triangular region with vertices (0, 0), (1, 0), and (1, 2), and let V be the volume of the solid generated when T is rotated about the line $x = a$, where $a > 1$. Express a in terms of V.

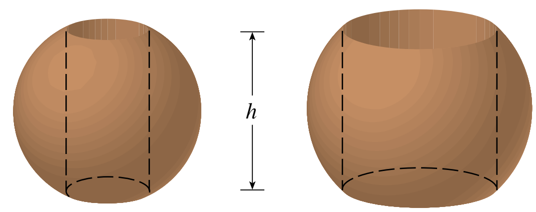
45–47 Use cylindrical shells to find the volume of the solid.

45. A sphere of radius r

46. The solid torus of Exercise 61 in Section 6.2

47. A right circular cone with height h and base radius r

48. Suppose you make napkin rings by drilling holes with different diameters through two wooden balls (which also have different diameters). You discover that both napkin rings have the same height h, as shown in the figure.
(a) Guess which ring has more wood in it.
(b) Check your guess: Use cylindrical shells to compute the volume of a napkin ring created by drilling a hole with radius r through the center of a sphere of radius R and express the answer in terms of h.



6.4 Work

The term *work* is used in everyday language to mean the total amount of effort required to perform a task. In physics it has a technical meaning that depends on the idea of a *force*. Intuitively, you can think of a force as describing a push or pull on an object—for example, a horizontal push of a book across a table or the downward pull of the earth's gravity on a ball. In general, if an object moves along a straight line with position function $s(t)$, then the **force** F on the object (in the same direction) is given by Newton's Second Law of Motion as the product of its mass m and its acceleration:

$$F = m\frac{d^2s}{dt^2} \tag{1}$$

In the SI metric system, the mass is measured in kilograms (kg), the displacement in meters (m), the time in seconds (s), and the force in newtons ($\mathrm{N} = \mathrm{kg{\cdot}m/s^2}$). Thus a force of 1 N acting on a mass of 1 kg produces an acceleration of 1 $\mathrm{m/s^2}$. In the US Customary system the fundamental unit is chosen to be the unit of force, which is the pound.

In the case of constant acceleration, the force F is also constant and the work done is defined to be the product of the force F and the distance d that the object moves:

$$W = Fd \qquad \text{work} = \text{force} \times \text{distance} \tag{2}$$

If F is measured in newtons and d in meters, then the unit for W is a newton-meter, which is called a joule (J). If F is measured in pounds and d in feet, then the unit for W is a foot-pound (ft-lb), which is about 1.36 J.

V EXAMPLE 1
(a) How much work is done in lifting a 1.2-kg book off the floor to put it on a desk that is 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8\ \mathrm{m/s^2}$.
(b) How much work is done in lifting a 20-lb weight 6 ft off the ground?

SOLUTION
(a) The force exerted is equal and opposite to that exerted by gravity, so Equation 1 gives

$$F = mg = (1.2)(9.8) = 11.76\ \mathrm{N}$$