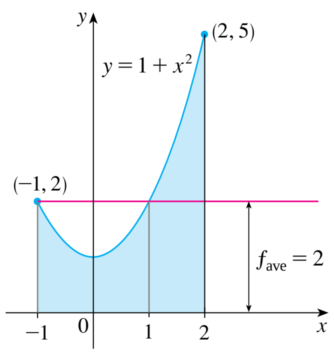


FIGURE 3

In this particular case we can find c explicitly. From Example 1 we know that $f_{ave} = 2$, so the value of c satisfies

$$f(c) = f_{ave} = 2$$

Therefore $\qquad 1 + c^2 = 2 \qquad \text{so} \qquad c^2 = 1$

So in this case there happen to be two numbers $c = \pm 1$ in the interval $[-1, 2]$ that work in the Mean Value Theorem for Integrals.

Examples 1 and 2 are illustrated by Figure 3.

V EXAMPLE 3 Show that the average velocity of a car over a time interval $[t_1, t_2]$ is the same as the average of its velocities during the trip.

SOLUTION If $s(t)$ is the displacement of the car at time t, then, by definition, the average velocity of the car over the interval is

$$\frac{\Delta s}{\Delta t} = \frac{s(t_2) - s(t_1)}{t_2 - t_1}$$

On the other hand, the average value of the velocity function on the interval is

$$\begin{aligned} v_{ave} &= \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} v(t)\,dt = \frac{1}{t_2 - t_1}\int_{t_1}^{t_2} s'(t)\,dt \\ &= \frac{1}{t_2 - t_1}[s(t_2) - s(t_1)] \qquad \text{(by the Net Change Theorem)} \\ &= \frac{s(t_2) - s(t_1)}{t_2 - t_1} = \text{average velocity} \end{aligned}$$

6.5 Exercises

1–8 Find the average value of the function on the given interval.

1. $f(x) = 4x - x^2, \quad [0, 4]$

2. $f(x) = \sin 4x, \quad [-\pi, \pi]$

3. $g(x) = \sqrt[3]{x}, \quad [1, 8]$

4. $g(t) = \dfrac{t}{\sqrt{3 + t^2}}, \quad [1, 3]$

5. $f(t) = e^{\sin t}\cos t, \quad [0, \pi/2]$

6. $f(\theta) = \sec^2(\theta/2), \quad [0, \pi/2]$

7. $h(x) = \cos^4 x \sin x, \quad [0, \pi]$

8. $h(u) = (3 - 2u)^{-1}, \quad [-1, 1]$

9–12
(a) Find the average value of f on the given interval.
(b) Find c such that $f_{ave} = f(c)$.
(c) Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

9. $f(x) = (x - 3)^2, \quad [2, 5]$

10. $f(x) = 1/x, \quad [1, 3]$

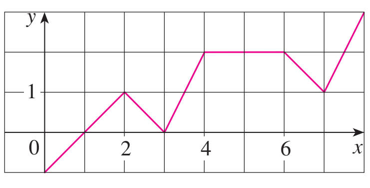
11. $f(x) = 2\sin x - \sin 2x, \quad [0, \pi]$

12. $f(x) = 2x/(1 + x^2)^2, \quad [0, 2]$

13. If f is continuous and $\int_1^3 f(x)\,dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

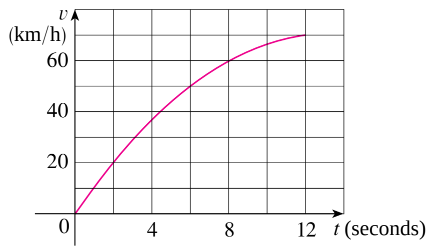
14. Find the numbers b such that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

15. Find the average value of f on $[0, 8]$.



Graphing calculator or computer required **1.** Homework Hints available at stewartcalculus.com

16. The velocity graph of an accelerating car is shown.



(a) Use the Midpoint rule to estimate the average velocity of the car during the first 12 seconds.
(b) At what time was the instantaneous velocity equal to the average velocity?

17. In a certain city the temperature (in °F) t hours after 9 AM was modeled by the function

$$T(t) = 50 + 14 \sin \frac{\pi t}{12}$$

Find the average temperature during the period from 9 AM to 9 PM.

18. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood (see Example 7 in Section 3.7). Find the average velocity (with respect to r) over the interval $0 \leqslant r \leqslant R$. Compare the average velocity with the maximum velocity.

19. The linear density in a rod 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.

20. (a) A cup of coffee has temperature 95°C and takes 30 minutes to cool to 61°C in a room with temperature 20°C. Use Newton's Law of Cooling (Section 3.8) to show that the temperature of the coffee after t minutes is

$$T(t) = 20 + 75e^{-kt}$$

where $k \approx 0.02$.
(b) What is the average temperature of the coffee during the first half hour?

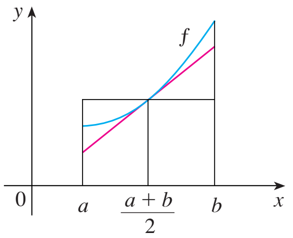
21. In Example 1 in Section 3.8 we modeled the world population in the second half of the 20th century by the equation $P(t) = 2560e^{0.017185t}$. Use this equation to estimate the average world population during this time period.

22. If a freely falling body starts from rest, then its displacement is given by $s = \frac{1}{2}gt^2$. Let the velocity after a time T be v_T. Show that if we compute the average of the velocities with respect to t we get $v_{\text{ave}} = \frac{1}{2}v_T$, but if we compute the average of the velocities with respect to s we get $v_{\text{ave}} = \frac{2}{3}v_T$.

23. Use the result of Exercise 83 in Section 5.5 to compute the average volume of inhaled air in the lungs in one respiratory cycle.

24. Use the diagram to show that if f is concave upward on $[a, b]$, then

$$f_{\text{ave}} > f\left(\frac{a+b}{2}\right)$$



25. Prove the Mean Value Theorem for Integrals by applying the Mean Value Theorem for derivatives (see Section 4.2) to the function $F(x) = \int_a^x f(t)\, dt$.

26. If $f_{\text{ave}}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{\text{ave}}[a, b] = \frac{c-a}{b-a} f_{\text{ave}}[a, c] + \frac{b-c}{b-a} f_{\text{ave}}[c, b]$$