It's clear that both systems must have expanded $(x^2 + 5)^8$ by the Binomial Theorem and then integrated each term.

If we integrate by hand instead, using the substitution $u = x^2 + 5$, we get

Derive and the TI-89 and TI-92 also give this answer.

$$\int x(x^2 + 5)^8\,dx = \tfrac{1}{18}(x^2 + 5)^9 + C$$

For most purposes, this is a more convenient form of the answer.

EXAMPLE 7 Use a CAS to find $\int \sin^5 x \cos^2 x\,dx$.

SOLUTION In Example 2 in Section 7.2 we found that

$$\boxed{1} \qquad \int \sin^5 x \cos^2 x\,dx = -\tfrac{1}{3}\cos^3 x + \tfrac{2}{5}\cos^5 x - \tfrac{1}{7}\cos^7 x + C$$

Derive and Maple report the answer

$$-\tfrac{1}{7}\sin^4 x \cos^3 x - \tfrac{4}{35}\sin^2 x \cos^3 x - \tfrac{8}{105}\cos^3 x$$

whereas Mathematica produces

$$-\tfrac{5}{64}\cos x - \tfrac{1}{192}\cos 3x + \tfrac{3}{320}\cos 5x - \tfrac{1}{448}\cos 7x$$

We suspect that there are trigonometric identities which show that these three answers are equivalent. Indeed, if we ask Derive, Maple, and Mathematica to simplify their expressions using trigonometric identities, they ultimately produce the same form of the answer as in Equation 1.

7.6 Exercises

1–4 Use the indicated entry in the Table of Integrals on the Reference Pages to evaluate the integral.

1. $\int_0^{\pi/2} \cos 5x \cos 2x\,dx$; entry 80

2. $\int_0^1 \sqrt{x - x^2}\,dx$; entry 113

3. $\int_1^2 \sqrt{4x^2 - 3}\,dx$; entry 39

4. $\int_0^1 \tan^3(\pi x/6)\,dx$; entry 69

5–32 Use the Table of Integrals on Reference Pages 6–10 to evaluate the integral.

5. $\int_0^{\pi/8} \arctan 2x\,dx$

6. $\int_0^2 x^2\sqrt{4 - x^2}\,dx$

7. $\int \dfrac{\cos x}{\sin^2 x - 9}\,dx$

8. $\int \dfrac{\ln(1 + \sqrt{x})}{\sqrt{x}}\,dx$

9. $\int \dfrac{dx}{x^2\sqrt{4x^2 + 9}}$

10. $\int \dfrac{\sqrt{2y^2 - 3}}{y^2}\,dy$

11. $\int_{-1}^0 t^2 e^{-t}\,dt$

12. $\int x^2 \operatorname{csch}(x^3 + 1)\,dx$

13. $\int \dfrac{\tan^3(1/z)}{z^2}\,dz$

14. $\int \sin^{-1}\sqrt{x}\,dx$

15. $\int e^{2x}\arctan(e^x)\,dx$

16. $\int x\sin(x^2)\cos(3x^2)\,dx$

17. $\int y\sqrt{6 + 4y - 4y^2}\,dy$

18. $\int \dfrac{dx}{2x^3 - 3x^2}$

19. $\int \sin^2 x \cos x \ln(\sin x)\,dx$

20. $\int \dfrac{\sin 2\theta}{\sqrt{5 - \sin\theta}}\,d\theta$

21. $\int \dfrac{e^x}{3 - e^{2x}}\,dx$

22. $\int_0^2 x^3\sqrt{4x^2 - x^4}\,dx$

23. $\int \sec^5 x\,dx$

24. $\int \sin^6 2x\,dx$

CAS Computer algebra system required   **1.** Homework Hints available at stewartcalculus.com

25. $\displaystyle\int \frac{\sqrt{4 + (\ln x)^2}}{x}\, dx$

26. $\displaystyle\int_0^1 x^4 e^{-x}\, dx$

27. $\displaystyle\int \frac{\cos^{-1}(x^{-2})}{x^3}\, dx$

28. $\displaystyle\int (t + 1)\sqrt{t^2 - 2t - 1}\, dt$

29. $\displaystyle\int \sqrt{e^{2x} - 1}\, dx$

30. $\displaystyle\int e^t \sin(\alpha t - 3)\, dt$

31. $\displaystyle\int \frac{x^4\, dx}{\sqrt{x^{10} - 2}}$

32. $\displaystyle\int \frac{\sec^2\theta \tan^2\theta}{\sqrt{9 - \tan^2\theta}}\, d\theta$

33. The region under the curve $y = \sin^2 x$ from 0 to π is rotated about the x-axis. Find the volume of the resulting solid.

34. Find the volume of the solid obtained when the region under the curve $y = \arcsin x$, $x \geqslant 0$, is rotated about the y-axis.

35. Verify Formula 53 in the Table of Integrals (a) by differentiation and (b) by using the substitution $t = a + bu$.

36. Verify Formula 31 (a) by differentiation and (b) by substituting $u = a \sin\theta$.

CAS **37–44** Use a computer algebra system to evaluate the integral. Compare the answer with the result of using tables. If the answers are not the same, show that they are equivalent.

37. $\displaystyle\int \sec^4 x\, dx$

38. $\displaystyle\int \csc^5 x\, dx$

39. $\displaystyle\int x^2\sqrt{x^2 + 4}\, dx$

40. $\displaystyle\int \frac{dx}{e^x(3e^x + 2)}$

41. $\displaystyle\int \cos^4 x\, dx$

42. $\displaystyle\int x^2\sqrt{1 - x^2}\, dx$

43. $\displaystyle\int \tan^5 x\, dx$

44. $\displaystyle\int \frac{1}{\sqrt{1 + \sqrt[3]{x}}}\, dx$

CAS **45.** (a) Use the table of integrals to evaluate $F(x) = \int f(x)\, dx$, where

$$f(x) = \frac{1}{x\sqrt{1 - x^2}}$$

What is the domain of f and F?

(b) Use a CAS to evaluate $F(x)$. What is the domain of the function F that the CAS produces? Is there a discrepancy between this domain and the domain of the function F that you found in part (a)?

CAS **46.** Computer algebra systems sometimes need a helping hand from human beings. Try to evaluate

$$\int (1 + \ln x)\sqrt{1 + (x \ln x)^2}\, dx$$

with a computer algebra system. If it doesn't return an answer, make a substitution that changes the integral into one that the CAS *can* evaluate.

DISCOVERY PROJECT CAS PATTERNS IN INTEGRALS

In this project a computer algebra system is used to investigate indefinite integrals of families of functions. By observing the patterns that occur in the integrals of several members of the family, you will first guess, and then prove, a general formula for the integral of any member of the family.

1. (a) Use a computer algebra system to evaluate the following integrals.

(i) $\displaystyle\int \frac{1}{(x + 2)(x + 3)}\, dx$ (ii) $\displaystyle\int \frac{1}{(x + 1)(x + 5)}\, dx$

(iii) $\displaystyle\int \frac{1}{(x + 2)(x - 5)}\, dx$ (iv) $\displaystyle\int \frac{1}{(x + 2)^2}\, dx$

(b) Based on the pattern of your responses in part (a), guess the value of the integral

$$\int \frac{1}{(x + a)(x + b)}\, dx$$

if $a \neq b$. What if $a = b$?

(c) Check your guess by asking your CAS to evaluate the integral in part (b). Then prove it using partial fractions.

CAS Computer algebra system required